$$\begin{cases} x+y+az=0 \\ ax-y=-1 \\ x+4y+6z=0 \end{cases}$$

$$\begin{vmatrix} 1 & 1 & a \\ a & -1 & 0 \\ 1 & 4 & 6 \end{vmatrix} = 0$$

$$-(6+0+4a^2)-(-a+0+6a)=0$$

$$4a^2-5a-6=0$$

$$a=\frac{5\pm 11}{8} \begin{cases} 2 \\ -3/4 \end{cases} \qquad 4(a-2)(a+3/4)$$

CASO I $a=2$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 2 & -1 & 0 & -1 \\ 1 & 4 & 6 & 0 \end{pmatrix} \overset{-2}{\sim} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -3 & -4 & -1 \\ 0 & 3 & 4 & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -3 & -4 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Rango $A=2$, Rango $A^*=3$ $\Rightarrow$ R-F $\Rightarrow$ S.I. $\Rightarrow \nexists$ Sol

nov 22-14:04

CASO II $a=3/4$

$$\begin{pmatrix} 1 & 1 & -3/4 & 0 \\ -3/4 & -1 & 0 & -1 \\ 1 & 4 & 6 & 0 \end{pmatrix} \overset{3}{\underset{4}{\sim}} \begin{pmatrix} 4 & 4 & -3 & 0 \\ -3 & -4 & 0 & -4 \\ 1 & 4 & 6 & 0 \end{pmatrix}$$

$$\overset{3}{\sim} \begin{pmatrix} 4 & 4 & -3 & 0 \\ 0 & -4 & -9 & -16 \\ 0 & 12 & 27 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & -3 & 0 \\ 0 & -4 & -9 & -16 \\ 0 & 0 & 0 & -48 \end{pmatrix}$$

Rango $A=2$, Rango $A^*=3$ $\Rightarrow$ R-F $\Rightarrow$ S.I. $\Rightarrow \nexists$ Sol

nov 22-14:11

CASO III $a \in \mathbb{R}-\{2,-3/4\}$

Nº ec = Nº incog $\Rightarrow$ CRAMER $\Rightarrow$ SCD $\Rightarrow \exists!$ Sol

$|A| \neq 0$

$$x=\frac{\begin{vmatrix} 0 & 1 & a \\ -1 & -1 & 0 \\ 0 & 4 & 6 \end{vmatrix}}{|A|}=\frac{6-4a}{4(a-2)(a+3/4)}$$

$$=\frac{3-2a}{2(a-2)(a+3/4)}$$

$$y=\frac{\begin{vmatrix} 1 & 0 & a \\ a & -1 & 0 \\ 1 & 0 & 6 \end{vmatrix}}{|A|}=\frac{-(6-a)}{4(a-2)(a+3/4)}=\frac{a-6}{4a^2-5a-6}$$

$$z=\frac{\begin{vmatrix} 1 & 1 & 0 \\ a & -1 & -1 \\ 1 & 4 & 0 \end{vmatrix}}{|A|}=\frac{3}{4a^2-5a-6}$$

Sol $(x,y,z)=(\ ;\ ;\ )$

nov 22-14:14

$$\begin{cases} x+y=7 \\ kx-y=11 \\ x-4y=k \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 7 \\ 1 & -4 & k \\ k & -1 & 11 \end{pmatrix} \overset{4}{\sim} \begin{pmatrix} y & x & \\ 1 & 1 & 7 \\ -4 & 1 & k \\ -1 & k & 11 \end{pmatrix}$$

$$\overset{\cdot}{\sim} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 5 & k+28 \\ 0 & k+1 & 18 \end{pmatrix} \overset{-5}{\sim} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 5 & k+28 \\ 0 & 0 & k^2+29k-62 \end{pmatrix}$$

$F_3=-5F_3+(k+1)F_2$

$$k^2+29k-62=0 \qquad k=\frac{-29\pm 33}{2} \begin{cases} 2 \\ -31 \end{cases}$$

nov 22-14:19

CASO I $k=2$

$$\left(\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 5 & 30 \\ 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{l} RA=2 \\ RA^*=2 \\ N^o\ Incog=2 \end{array}\right] \xRightarrow{R-F} SCD \quad \exists!\ Sol$$

$y+x=7 \quad y=1 \quad Sol: (x,y)=(6,1)$

$5x=30 \to x=6$

CASO II $k=-3$

$$\left(\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 5 & -3 \\ 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{l} RA=2 \\ RA^*=2 \\ N\ Incog=2 \end{array}\right] \xRightarrow{R-F}$$

$SCD \Rightarrow \exists!\ Sol$

$y+x=7$

$5x=-3 \quad x=-3/5 \quad (x,y)=(-3/5, 38/5)$

$y=38/5$

nov 22-14:22

CASO III

$$\left(\begin{array}{cc|c} 1 & 1 & 7 \\ 0 & 5 & * \\ 0 & 0 & * \neq 0 \end{array}\right) \quad \left.\begin{array}{l} RA=2 \\ RA^*=3 \end{array}\right] \xRightarrow{R-F}$$

$SI \Rightarrow \nexists\ Sol$

nov 22-14:25

$$\begin{cases} (a-1)x+y=0 \\ (a-1)x+(a+1)y=0 \end{cases}$$

$$\begin{vmatrix} a-1 & 1 \\ a-1 & a+1 \end{vmatrix} = a^2-1-(a-1) = a^2-a=0$$

$a(a-1)=0 \begin{cases} a=0 \\ a=1 \end{cases}$

CASO I $a=0$

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{l} RA=1 \\ RA^*=1 \\ N^o\ Incog=2 \end{array}\right]$$

$$\xRightarrow{R-F} \left[\begin{array}{l} SCI \Rightarrow \exists\ \text{inf sol} \\ -x+y=0 \quad y=\alpha,\ x=\alpha \quad (x,y)=(\alpha,\alpha)\ \forall \alpha \in \mathbb{R} \end{array}\right.$$

nov 22-14:28

CASO II $a=1$

$$\left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 2 & 0 \end{array}\right) \sim \begin{array}{c} \begin{array}{cc} y & x \end{array} \\ \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 2 & 0 & 0 \end{array}\right) \end{array} \sim$$

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{l} RA=1 \\ RA^*=1 \\ N^o\ I=2 \end{array}\right] \xRightarrow[\text{inf}]{R-F} SCI \Rightarrow \exists\ Sol$$

$y=0$

$x=\alpha \quad (x,y)=(\alpha,0)\ \forall \alpha \in \mathbb{R}$

nov 22-14:31

CASO III $a \in \mathbb{R} - \{0, 1\}$

$\left.\begin{array}{l} \text{Nº ec} = \text{Nº I vcos} \\ |A| \neq 0 \end{array}\right\} \overset{\text{Cramer}}{\Rightarrow} \text{SCD} \Rightarrow \exists!\ \text{sol}$

Homogéneo $\Rightarrow (x, y) = (0, 0)$

nov 22-14:34

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(1) $AX = B \Rightarrow \underbrace{\underbrace{A^{-1}A}_{I}X}_{} = A^{-1}B$

$\boxed{X = A^{-1}B}$

(2) $\left\{\begin{array}{l} x_1 + 2x_1 + 3x_3 = 1 \\ \vdots \end{array}\right\}$ Sist

nov 22-14:36

RANGO DE UNA MATRIZ CON PARAM

- Matriz cuadrada

$|A| = 0$

- Matriz cuadrada
  - El param $\downarrow \rightarrow$
  - Hacer ceros
  - Igualar los elementos de la diagonal pa separado a cero

Valores del parámetro → Un caso más que valores del param

nov 22-14:38

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & a & -3 \\ 4 & 1 & a \end{pmatrix} \quad \text{¿Rango } A\text{?}$$

$|A| = a^2 - (-4a - 3) = a^2 + 4a + 3$

$|A| = 0 \quad a = \dfrac{-4 \pm 2}{2} \begin{array}{l} -1 \\ -3 \end{array}$

CASO I $a = -1$

$$-4\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & -3 \\ 4 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & -3 \\ 0 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

Rango $A = 2$

CASO II $a = -3$

nov 22-14:40

$$\begin{pmatrix}1&0&-1\\0&-3&-3\\4&1&-3\end{pmatrix}\sim\begin{pmatrix}1&0&-1\\0&-3&-3\\0&1&1\end{pmatrix}\sim\begin{pmatrix}1&0&-1\\0&3&-3\\0&0&0\end{pmatrix}$$

Rango $A=2$

Caso III $a\in\mathbb{R}-\{-1,-3\}$

$|A|\neq 0 \rightarrow$ Rango $A=3$

nov 22-14:42

$$\left.\begin{array}{l}AB=A\\BA=B\end{array}\right]\Longrightarrow ¿A^2=A?$$

$AB=A$

$AAB=AA$

$A^2=A^2$

$AB=A$

$(AB)A=AA$

$A(BA)=A^2$

$AB=A^2$

$A=A^2$

nov 22-14:46

$$|C_1\ C_2\ C_3|=6$$

$$|3C_1-C_3\ \ 2C_3\ \ C_2|$$

$$2\,|3C_1-C_3\ \ C_3\ \ C_2|$$

$C_1=C_1+C_2$

$$2\,|3C_1\ \ C_3\ \ C_2|=6|C_1\ C_3\ C_2|=$$

$C_2\leftrightarrow C_3$

$$=-6|C_1\ C_2\ C_3|=-6\cdot 6=-36$$

nov 22-14:51