TEMA 1 Sistemas

Tª: Clasif de los sistemas

Sistema homogeneo

Cuestiones Clasif + Resol Sistema

Problemas

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TEMA 2 - MATRICES

Tª: Tipos + Ejemplos

Cuestiones

Operaciones: $+, -, \cdot, A^{-1}, A^{n}, \ldots$

Tipos de ejercicios

Ecuaciones: $AX+B=C \quad AX=C-B$

$A^{-1}AX=A^{-1}(C-B) \quad X=A^{-1}(C-B)$

Sistemas $\begin{cases} AX+BY=C \\ A'X+B'Y=C' \end{cases}$ → Resol $X=$ $Y=$ Hacer Op.

Conmutan $AX=XA$ → Sistema C.I ⇒ Resolver

Potencias $A, A^2, A^3, A^4 \ldots A^n$ $\begin{cases} \text{Formula} \\ n \quad \text{Rep} \\ R \end{cases}$

Inversa $\begin{cases} \text{DEF } AA^{-1}=I \\ \text{GAUSS} \Rightarrow (A|I) \sim (I|A^{-1}) \\ \text{DET } A^{-1}=\frac{1}{|A|}(\text{Adj } A)^t \end{cases}$

Rangos ⇒ GAUSS ⇒ Nº lineas ≠ 0

Problemas

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TEMA 3 - DETERMINANTES

Tª - Propiedades

$|A|=0$ $\begin{cases} L_i=0 \\ L_i=L_j \\ L_i=\alpha L_j \\ L_i=\alpha L_j+\beta L_k \end{cases}$

$|AB|=|A|\cdot|B|$

$|A+B|\neq|A|+|B| \quad \begin{vmatrix} a & c \\ b & d \end{vmatrix}+\begin{vmatrix} a & c' \\ b & d' \end{vmatrix}=\begin{vmatrix} a & c+c' \\ b & d+d' \end{vmatrix}$

$|A^t|=|A|$

$|\alpha A|=\alpha^n|A|$

$|A^{-1}|=\frac{1}{|A|}$

$F_1 \leftrightarrow F_2 \quad |A|$ Cambio de signo

$F_2=\alpha F_2+2F_1 \quad \frac{1}{\alpha}|\quad|$

$\begin{vmatrix} a & d & e \\ 0 & b & F \\ 0 & 0 & c \end{vmatrix}=abc$

Cálculo

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$

$\begin{vmatrix} a & b & c \\ d & e & F \\ g & h & i \end{vmatrix}=[\diagdown+\triangle+\triangledown]-[\diagup+\triangle+\triangledown]$

Orden>3 $\begin{vmatrix} a & b & c & d \\ 0 & e & F & g \\ 0 & h & i & j \\ 0 & k & l & m \end{vmatrix}=(-1)^{i+j}a\begin{vmatrix} \square \end{vmatrix}$

$\begin{vmatrix} X & 3 & 3 & 3 \\ 3 & X & 3 & 3 \\ 3 & 3 & X & 3 \\ 3 & 3 & 3 & X \end{vmatrix}=$ $C_1+C_2+C_3+C_4$

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$\exists A^{-1}$

$|A|=0 \quad \nexists A^{-1} \Rightarrow A$ singular

$|A|\neq 0 \quad \exists A^{-1} \Rightarrow A$ regular

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TEMA 4 Resol. del sist y Rangos

Tª { Regla de Cramer
R-F

SISTEMAS

No cuadrada
Nº ec ≠ Nº incog
Param ↓ →
Ceros
elem diag = 0

Cuadrada
Nº ec = Nº incog
$|A| = 0 \rightarrow$

Valores del param
↓
1 caso mas

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Sistema

$$\begin{cases} a_{11}x_1 + \dots + a_{1n} = b_1 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn} = b_m \end{cases}$$

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$AX = B \rightarrow$ Ec. matricial

$$\begin{pmatrix} k & 1 & 2 \\ 3 & 4 & 5 \\ 2 & k & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ k \\ 3 \end{pmatrix}$$

$$\begin{cases} kx + y + 1z = 2 \\ \vdots \end{cases}$$

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(1) $\begin{cases} x + ay + z = a + 2 \\ x + y + az = -2(a+1) \\ ax + y + z = a \end{cases}$

$$\begin{vmatrix} 1 & a & 1 \\ 1 & 1 & a \\ a & 1 & 1 \end{vmatrix} = |1 + a^3 + 1| - (a + a + a)$$

$a^3 - 3a + 8 = 0$

$$\begin{array}{c|cccc} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$a = \frac{-1 \pm 3}{8}$ $\begin{cases} 1 \\ -2 \end{cases}$

$\boxed{a = 1 \text{ doble}}$
$\boxed{a = -2}$

CASO I $a = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -7 \\ 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\left.\begin{array}{l} RA = 1 \\ RA^* = 2 \end{array}\right] \xRightarrow{R-F}$ Sist. Incompatible $\Rightarrow \nexists$ Sol

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CASO II $a = -2$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 2 \\ -2 & 1 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 2 \\ 0 & -3 & 3 & -8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \left.\begin{array}{l} RA = 2 \\ RA^* = 2 \\ \text{Nº Incog } 3 \end{array}\right] \xRightarrow{RF}$$

Sist. Comp Ind $\Rightarrow \exists$ ~~un~~ F sol

CASO III $a \in \mathbb{R} - \{1, -8\}$

$\left.\begin{array}{l} \text{Nº ec} = \text{Nº incog} \\ |A| \neq 0 \end{array}\right] \xRightarrow{\text{Cramer}}$ Sist Comp Det $\Rightarrow \exists!$ Sol

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b) $a=-1 \rightarrow$ SCD $\Rightarrow$ ¡! Sol

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & -1 \\ 0 & 0 & 2 & 0 \end{array}\right)$$

$x-y+z=1 \quad x=1/2$

$2y-2z=-1 \quad y=-1/2$

$2z=0 \quad z=0$

$(x,y,z)=\left(1/2,-1/2,0\right)$

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2) $\begin{cases} x+2y+z=3 \\ x+3y+2z=5 \\ x+my+3z=7 \end{cases}$

$|A|=\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & m & 3 \end{vmatrix}=4-m \quad m=4$

CASO I $m=4$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 3 & 2 & 5 \\ 1 & 4 & 3 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$RA=2$, $RA^*=2$, N° Incog $=3$ $\Rightarrow$ R-F $\Rightarrow$ SCI $\Rightarrow$ Infsol

CASO II $m\neq 4$

N° Ecs $=N^{\circ}$ Incog, $|A|\neq 0$ $\Rightarrow$ Cramer $\Rightarrow$ SCD $\Rightarrow$ Sol

SOL Siempre compatible

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Resolver

$m=4$ $x+2y+z=3$ $x=\alpha-1$

$y+z=2$ $y=2-\alpha$

$z=\alpha$

Sol $(\alpha-1, 2-\alpha, \alpha) \ \forall \alpha \in \mathbb{R}$

$m\neq 4$:

$$x=\frac{\begin{vmatrix} 3 & 2 & 1 \\ 5 & 1 & 2 \\ 7 & m & 3 \end{vmatrix}}{|A|}=\frac{4-m}{4-m}=1$$

$$y=\frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 5 & 2 \\ 1 & 7 & 3 \end{vmatrix}}{|A|}=\frac{0}{4-m}=0$$

$$z=\frac{\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & m & 7 \end{vmatrix}}{|A|}=\frac{8-2m}{4-m}=2$$

Sol: $(x,y,z)=(1,0,2)$

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3) $M=\begin{pmatrix} 2 & 1-a & a \\ 2a & 1 & -1 \\ 2 & a & 1 \end{pmatrix}$

$\exists M^{-1} \quad |M|\neq 0$

$|M|=2-2-2a^3+2a+2a-2a$

$-2a^3+2a=0$

$2a(-a^2+1)=0 \quad \begin{cases} a=0 \\ a=\pm 1 \end{cases}$

$a\in \mathbb{R}-\{0,-1,+1\}$

$A=\begin{pmatrix} 2 & 1 & -2 \\ 4 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix} \quad A^{-1}=\begin{pmatrix} -1/4 & 5/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/6 & 1/6 \end{pmatrix}$

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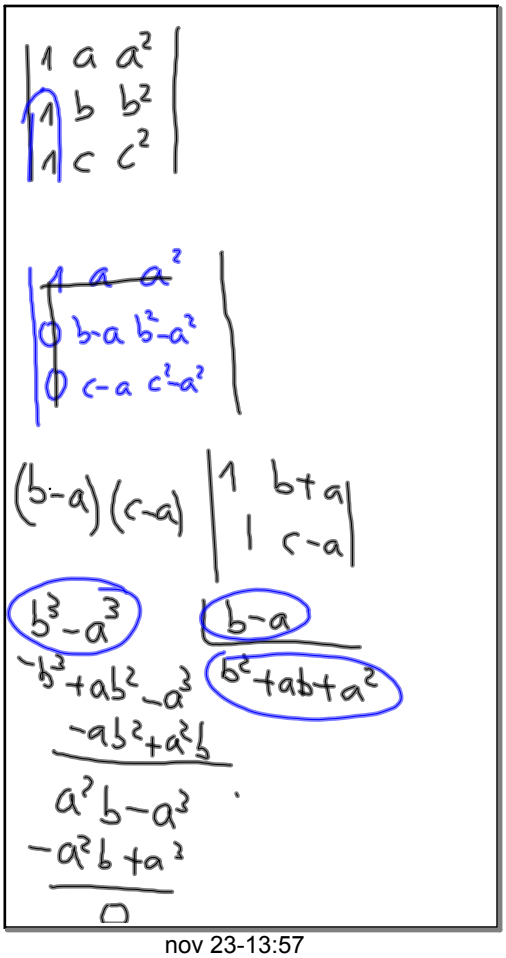
$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$
$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$
$(b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c-a \end{vmatrix}$
b^3-a^3
$b-a$
$-b^3+ab^2-a^3$
b^2+ab+a^2
$-ab^2+a^2b$
a^2b-a^3
$-a^2b+a^3$
0
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