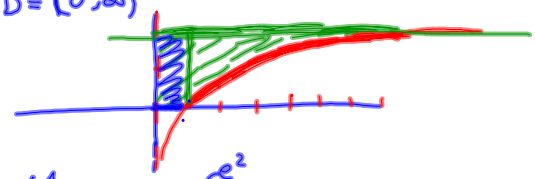
(33) $y=\ln x \quad y=2 \quad ox.oy$

$\begin{cases} y=\ln x \\ y=2 \end{cases} \quad \ln x=2 \quad e^2=x \quad (e^2,2)$

$y=\ln x$ $D=(0,\infty)$

x	0	1	2	3	4	5
y	$-\infty$	0	0'7	1'1	1'39	1'61



$\int_0^4 2\,dx + \int_1^{e^2} (2-\ln x)\,dx$

$y=2$ y OX ; $y=2$ $y=\ln x$

$I_1=\int_0^1 2\,dx=2x\Big]_0^1=2-0=2\,u^2$

$I_2=\int(2-\ln x)\,dx=(2-\ln x)x-\int x\left(-\frac{1}{x}\right)dx$

$\begin{cases} u=2-\ln x & du=-\frac{1}{x}dx \\ dx=dv & v=\int dx=x \end{cases}$ $(2-\ln x)x+x+C$ $(2-\ln x+1)x$

$(3-\ln x)x+C$

$I_2=(3-\ln x)x\Big]_1^{e^2}=$

$\left[(3-\ln e^2)e^2\right]-\left[(3-\ln 1)1\right]$

$(3-2)e^2-3=e^2-3\,u^2$

$I=I_1+I_2=2+e^2-3=e^2-1\,u^2$

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(38) $y=e^x$

$y=e^{2x}$

$x=k$

Area $k=1$

$\begin{cases} y=e^x \\ y=e^{2x} \end{cases} \quad e^{2x}=e^x \quad 2x=x \quad x=0 \quad (0,1)$

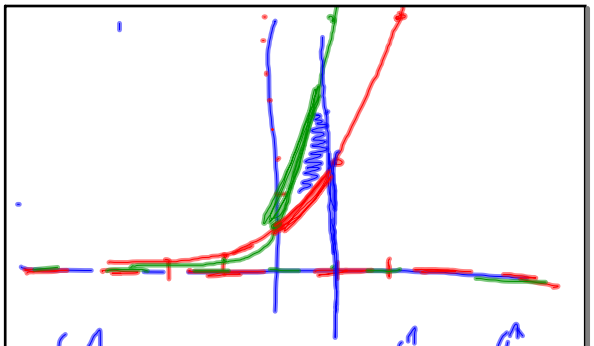
$y=e^x$

x	$-\infty$	-2	-1	0	1	2	$+\infty$
y	0	$\frac{1}{e^2}$	$\frac{1}{e}$	1	e	e^2	$+\infty$

$y=e^{2x}$

x	$-\infty$	-2	-1	0	1	2	$+\infty$
y	0	$\frac{1}{e^4}$	$\frac{1}{e^2}$	1	e^2	e^4	$+\infty$

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$\int_0^1 (e^{2x}-e^x)\,dx=\frac{1}{2}\int_0^1 e^{2x}\,dx-\int_0^1 e^x\,dx$

$\frac{1}{2}e^{2x}-e^x\Big]_0^1$

$\left[\left(\frac{1}{2}e^2-e\right)-\left(\frac{1}{2}e^0-e^0\right)\right]$

$\frac{1}{2}e^2-e-\frac{1}{2}+1$

$\frac{1}{2}e^2-e+\frac{1}{2}=\frac{e^2-2e+1}{2}\,u^2$

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Producto escalar (Número)

$\bar{u}\cdot\bar{v}=|\bar{u}|\cdot|\bar{v}|\cdot\cos(\hat{\bar{u}},\bar{v})$

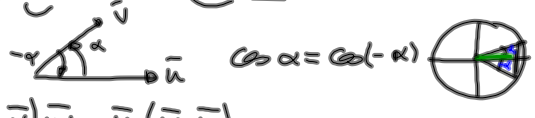
$\bar{u}=\bar{0} \quad \bar{0}\cdot\bar{v}=|\bar{0}|\,|\bar{v}|\cos(\bar{0},\bar{v})=0$

$\bar{u}\perp\bar{v}\Rightarrow\bar{u}\cdot\bar{v}=|\bar{u}|\cdot|\bar{v}|\underbrace{\cos(\bar{u},\bar{v})}_{90^\circ}=0$

Si $\bar{u},\bar{v}$ y ninguno nulo $\bar{u}\cdot\bar{v}=0\Leftrightarrow\bar{u}\perp\bar{v}$

$\bar{u}\cdot\bar{v}=\bar{v}\cdot\bar{u}$

$|\bar{u}|\cdot|\bar{v}|\cos(\bar{u},\bar{v})=|\bar{v}|\cdot|\bar{u}|\cos(\bar{v},\bar{u})$



$\cos\alpha=\cos(-\alpha)$

$(\bar{u}\cdot\bar{v})\cdot\bar{w}=\bar{u}\cdot(\bar{v},\bar{w})$

$a(\bar{v}\cdot\bar{w})=(a\bar{v})\cdot\bar{w}$

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Producto escalar $\bar{u}\cdot\bar{v} = |\bar{u}|\cdot|\bar{v}|\cdot\cos(\hat{u},\hat{v})$

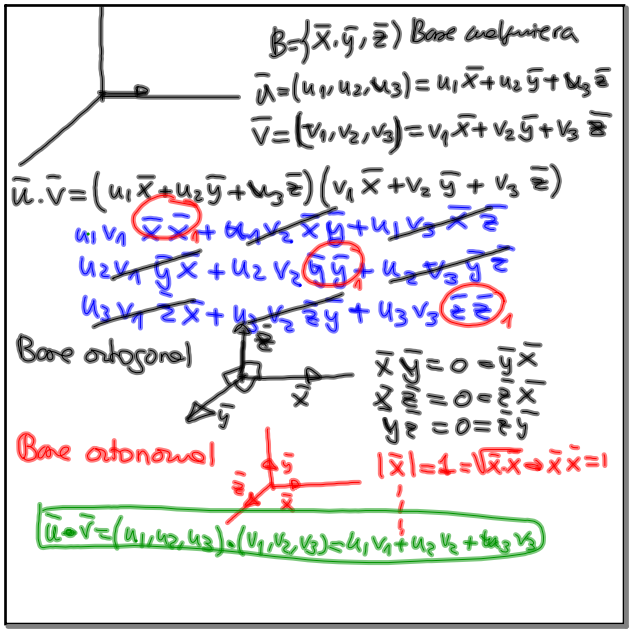
$\bar{u}\cdot\bar{u} = |\bar{u}|\cdot|\bar{u}|\cos(\bar{u},\bar{u}) \Rightarrow \bar{u}\cdot\bar{u} = |\bar{u}|^2$ ($\cos 0^\circ = 1$)

$|\bar{u}| = \sqrt{\bar{u}\cdot\bar{u}}$

Modulo de un vector $|\bar{u}| = \sqrt{\bar{u}\cdot\bar{u}}$

Angulo entre 2 vectores $\cos(\bar{u},\bar{v}) = \dfrac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$

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$$|\bar{u}| = \sqrt{\bar{u}\cdot\bar{u}} = \sqrt{(u_1,u_2,u_3)(u_1,u_2,u_3)} = \sqrt{u_1^2+u_2^2+u_3^2}$$

$$\cos(\bar{u},\bar{v}) = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}|\,|\bar{v}|} = \frac{u_1v_1+u_2v_2+u_3v_3}{\sqrt{u_1^2+u_2^2+u_3^2}\,\sqrt{v_1^2+v_2^2+v_3^2}}$$

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	EXP. VECTORIAL $\bar{u}, \bar{v}$	EXP ANALITICA $\bar{u}=(u_1,u_2,u_3)$ $\bar{v}=(v_1,v_2,v_3)$
Producto escalar (θ°)	$\bar{u}\cdot\bar{v} = \lvert\bar{u}\rvert\lvert\bar{v}\rvert\cos(\hat{u},\hat{v})$	$\bar{u}\cdot\bar{v} = u_1v_1+u_2v_2+u_3v_3$
Modulo	$\lvert\bar{v}\rvert = \sqrt{\bar{v}\cdot\bar{v}}$	$\lvert\bar{v}\rvert = \sqrt{v_1^2+v_2^2+v_3^2}$
Angulo	$\cos(\bar{u},\bar{v}) = \dfrac{\bar{u}\cdot\bar{v}}{\lvert\bar{u}\rvert\lvert\bar{v}\rvert}$	$\cos(\bar{u},\bar{v}) =$ ____
Vectores paralelos $\bar{u}\parallel\bar{v}$	$\bar{u} = \alpha\bar{v}$	$\dfrac{u_1}{v_1} = \dfrac{u_2}{v_2} = \dfrac{u_3}{v_3}$
Vectores perp o ortogonales $\bar{u}\perp\bar{v}$	$\bar{u}\cdot\bar{v} = 0$	$u_1v_1+u_2v_2+u_3v_3 = 0$
Conv. un vector en unitario	$\dfrac{\bar{v}}{\lvert\bar{v}\rvert}$	$\left(\dfrac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}}, \dfrac{v_2}{\quad}, \dfrac{v_3}{\quad}\right)$
Proyecc ortogonal	$\lvert\bar{u}'\rvert = \dfrac{\lvert\bar{u}\cdot\bar{v}\rvert}{\lvert\bar{v}\rvert}$	
Vector proyec ortogonal	$\bar{u}' = \dfrac{\lvert\bar{u}\cdot\bar{v}\rvert}{\lvert\bar{v}\rvert^2}\bar{v}$	$\bar{u} = \dfrac{\lvert u_1v_1+u_2v_2+u_3v_3\rvert}{v_1^2+v_2^2}(v_1,v_2,v_3)$

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Convertir en unitario

$(1,2,3)$

$|u| = \sqrt{1+4+9} = \sqrt{14}$

$\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right) \Rightarrow |\bar{w}| = \sqrt{\frac{1}{14}+\frac{4}{14}+\frac{9}{14}} = 1$

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Proyección ortogonal



$\cos(\bar{u}, \bar{v}') = \frac{|\bar{v}'|}{|\bar{v}|}$

$|\bar{v}'| = |\bar{v}| \cos(\bar{u}, \bar{v}') = |\bar{v}| \cos(\bar{u}, \bar{v})$

$\frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$

Proyección ortogonal $|\bar{v}'| = \frac{|\bar{u}\cdot\bar{v}|}{|\bar{u}|}$

Vector proyección $\bar{v}' = \frac{|\bar{u}\cdot\bar{v}|}{|\bar{u}|}\frac{\bar{u}}{|\bar{u}|} = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}|^2}\bar{u}$

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