$x^3+3x+1=0$ Demostrar $\exists!$ Solución

[1] Demostrar, existe una sol. de la ec: $x^3+3x+1=0$

$f(x)=x^3+3x+1$

$f(0)=1$, $f(-1)=-3$; f cont $[-1,0]$; signo $f(-1)\neq$ signo $f(0)$ $\Rightarrow \exists c\in(-1,0)$ / $f(c)=0$

$\Rightarrow \exists c\in(-1,0)$ que es sol de la ec. $x^3+3x+1=0$

[2] UNICIDAD (TM Rolle) por red. al absurdo

Suponemos que hay 2 soluciones $x=a \Rightarrow f(a)=0$, $x=b \Rightarrow f(b)=0$

$f(x)=x^3+3x+1$

f cont $[a,b]$, f der. (a,b), $f(a)=f(b)$ $\xrightarrow{\text{T Rolle}}$ $\exists c\in(a,b)$ / $f'(c)=0$

$f'(x)=3x^2+3=0$ $x=\pm\sqrt{-1}$ no tiene raíces

$\Rightarrow \exists!$ solución

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4j) 2) $\lim_{x\to\pi/2^-} \cos x \ln \operatorname{tg} x = \cos\frac{\pi}{2}\ln \operatorname{tg}\frac{\pi}{2} = 0\cdot\ln+\infty = 0\cdot+\infty$ (Ind) $\frac{1}{0^+}$

$\ln f\cdot g = \ln\frac{f}{1/g}$

$\lim_{x\to\pi/2^-} \frac{\ln \operatorname{tg} x}{\frac{1}{\cos x}} = \frac{+\infty}{+\infty} \overset{\text{L'H}}{=} \lim_{x\to\pi/2^-} \frac{\frac{1}{\operatorname{tg} x}\cdot\frac{1}{\cos^2 x}}{\frac{+\sin x}{\cos^2 x}} =$

$\lim_{x\to\pi/2^-} \frac{1}{\operatorname{tg} x\cdot\sin x} = \frac{1}{\operatorname{tg}\frac{\pi}{2}\cdot\sin\frac{\pi}{2}} = \frac{1}{+\infty\cdot 1} = 0$

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c) $\lim_{x\to\pi/2^-} (\operatorname{tg} x)^{\cos x} = (+\infty)^{0^+}$ $f(x)=e^{g(x)\ln f(x)}$

$\ln(\operatorname{tg} x)^{\cos x}$

$\lim_{x\to\pi/2^-} e^{\cos x \ln(\operatorname{tg} x)}$ Apart. a) $e^0 = 1$

$\lim_{x\to\pi/2^-} e$

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45) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right) = \frac{1}{0}-\frac{1}{0}$

$\lim_{x\to 0}\frac{x-\sin x}{x\sin x} = \frac{0}{0}$

$\overset{\text{L'H}}{\approx} \lim_{x\to 0}\frac{1-\cos x}{\sin x + x\cos x} = \frac{1-1}{0+0} = \frac{0}{0}$

$\overset{\text{L'H}}{\approx} \lim_{x\to 0}\frac{\sin x}{\cos x+\cos x+x(-\sin x)} =$

$\frac{0}{1+1-0} = \frac{0}{2} = 0$

$\approx \lim_{x\to 0}\frac{x-\sin x}{x^2} = \frac{0}{0}$

$\overset{\text{L'H}}{\approx} \lim_{x\to 0}\frac{1-\cos x}{2x} = \frac{1-1}{0-0} = \frac{0}{0}$

$\overset{\text{L'H}}{\approx} \lim_{x\to 0}\frac{\sin x}{2} = \frac{0}{2} = 0$

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c) $\lim_{x\to 1} \frac{e}{e^x - e} - \frac{1}{x-1}$

$\frac{e}{e-e} - \frac{1}{1-1} = \frac{e}{0} - \frac{1}{0}$

$\lim_{x\to 1} \frac{ex - e - e^x + e}{e^x x - e^x - ex + e} = \frac{0}{0}$

$\overset{L'H}{\sim} \lim_{x\to 1} \frac{e - e^x}{e^x x + e^x - e^x - e} = \frac{0}{0}$

$\overset{L'H}{\sim} \lim_{x\to 1} \frac{-e^x}{e^x x + e^x} =$

$\frac{-e}{e+e} = \frac{-e}{2e} = -\frac{1}{2}$

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(56) $f(x) = x^3 - 18x \quad [0, 3\sqrt{2}]$

Hipot. T^a Rolle

[1] f cont $[0, 3\sqrt{2}]$, cierto para f(x) es un polinomio y es cont en $\mathbb{R}$.

[2] f der en $(0, 3\sqrt{2})$

$f'(x) = 3x^2 - 18$. Dem v. en $\mathbb{R}$

[3] $f(0) = f(3\sqrt{2})$ $(\sqrt{2})^3 = \sqrt{2^3}$

$f(0) = 0 - 0 = 0$

$f(3\sqrt{2}) = 27\sqrt{8} - 18 \cdot 3\sqrt{2} = 54\sqrt{2} - 54\sqrt{8} = 0$ Cierto

Cumple hip. de/ Teorema de Rolle

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$\exists c \in (0, 3\sqrt{2}) \ni f'(c) = 0$

$f'(x) = 3x^2 - 18 = 0$

$x^2 = 6 \quad x = \pm\sqrt{6} \in (0, 3\sqrt{2})$

$\boxed{c = +\sqrt{6}}$

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(57) $f(x) = \begin{cases} \frac{1}{x} & -2 \le x \le -1 \\ \frac{x^2 - 3}{2} & -1 \le x \le 0 \end{cases}$

Hip. TVM $[-2, 0]$

[1] f cont $[-2, 0]$

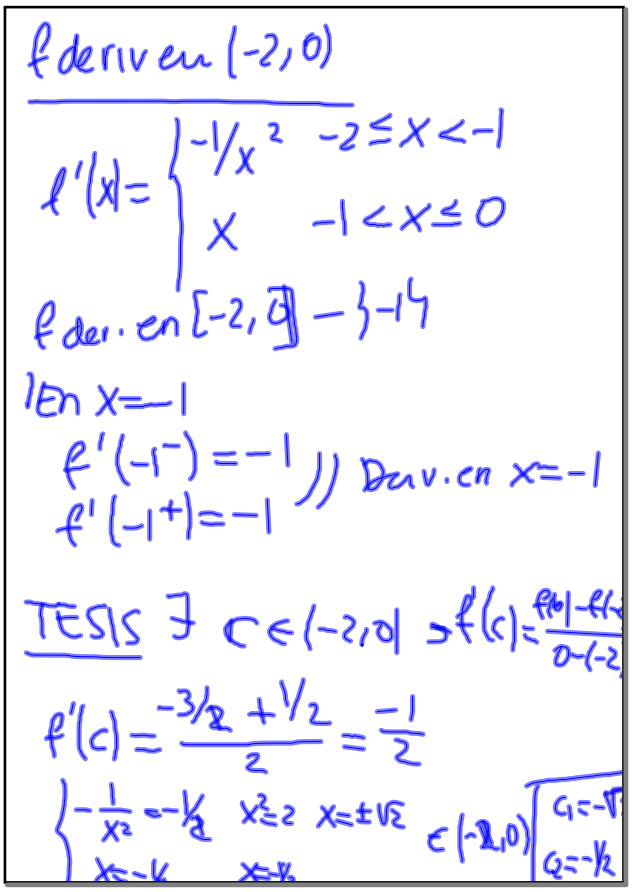
f cont $[-2, -1] - \{-1\}$

En $x = -1$

$\lim_{x\to -1} f(x)$ $\begin{cases} \lim_{x\to -1^-} \frac{1}{x} = -1 \\ \lim_{x\to -1^+} \frac{x^2 - 3}{2} = -1 \end{cases}$ f cont en $x = -1$

$f(-1) = -1$

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f deriv en (-2,0)
f'(x) = -1/x^2 -2 ≤ x < -1
x -1 < x ≤ 0
f der. en [-2,0] - {-1}
En x = -1
f'(-1⁻) = -1 // Deriv. en x = -1
f'(-1⁺) = -1
TEOR ∃ c ∈ (-2,0) / f'(c) =
f'(c) = (-3/2 + 1/2)/2 = -1/2
-1/x² = -1/2 x² = 2 x = ±√2
c ∈ (-2,0)
c1 = -√
c2 = -1/2

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