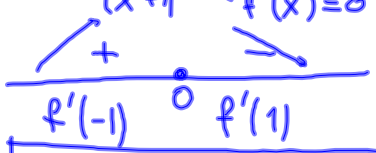
⑥ e) $y=\frac{1}{x^2+1}$

MONOT + EXT. RELATIVOS

$D(f): x^2+1\neq 0 \quad D=\mathbb{R}$

$f'(x)=\frac{-2x}{(x^2+1)^2} \quad D(f')=\mathbb{R}$

$f'(x)=0 \quad x=0$



Crec $(-\infty,0)$
Decr $(0,+\infty)$
Max rel $(0,f(0))=(0,1)$

ene 28-13:48

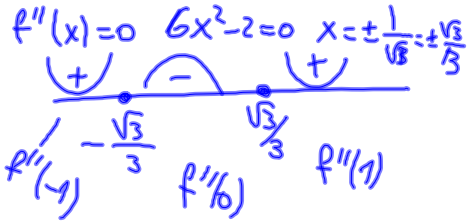
CURV Y PI

$f''(x)=\frac{-8(x^2+1)^2+2x\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$\frac{-2x^2-2+8x^2}{(x^2+1)^3}=\frac{6x^2-2}{(x^2+1)^3}$

$D(f\cap f'\cap f'')=\mathbb{R}$

$f''(x)=0 \quad 6x^2-2=0 \quad x=\pm\frac{1}{\sqrt{3}}=\pm\frac{\sqrt{3}}{3}$



Con eva $(-\sqrt{3}/3, \sqrt{3}/3)$
Convexa $(-\infty,-\sqrt{3}/3)\cup(\sqrt{3}/3,+\infty)$
PI $\left(\pm\frac{\sqrt{3}}{3},\frac{3}{4}\right)$

ene 28-14:08

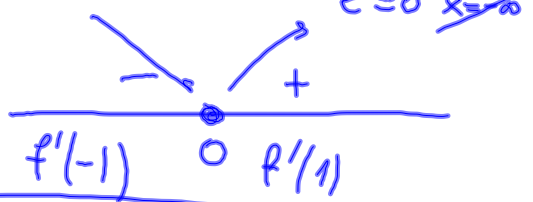
f) $y=e^x(x-1)$

MONOT + EXT

$y'=e^x(x-1)+e^x\cdot 1=xe^x$

$D(f\cap f')=\mathbb{R}$

$f'(x)=0 \quad xe^x=0 \begin{cases} x=0 \\ e^x=0 \quad x=-\infty \end{cases}$



Dec $(-\infty,0)$
Crec $(0,\infty)$
Mín $(0,f(0))=(0,-1)$

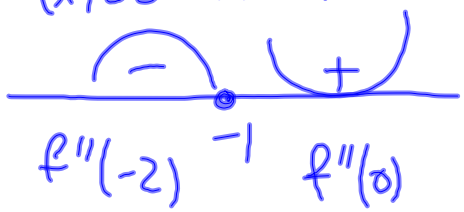
ene 28-14:11

CURV + PI

$y''=1\cdot e^x+xe^x=(1+x)e^x$

$D(f\cap f'\cap f'')=\mathbb{R}$

$f''(x)=0 \quad x=-1$



Concava $(-\infty,-1)$
Convexa $(-1,\infty)$
PI $(-1,f(-1))=\left(-1,-\frac{2}{e}\right)$

ene 28-14:13

7a) $y = \frac{8-3x}{x(x-2)} = \frac{8-3x}{x^2-2x}$

M + EXT. R

$y' = \frac{-3(x^2-2x)-(8-3x)(2x-2)}{(x^2-2x)^2}$

$\frac{-3x^2+\cancel{6x}-16x+16+6x^2-\cancel{6x}}{(x^2-2x)^2}$

$\frac{3x^2-16x+16}{(x^2-2x)^2}$

$D(f \cap f') = \mathbb{R} - \{0,2\}$

$f'(x) = 3x^2-16x+16=0$

$x = \frac{16 \pm 8}{6} < \begin{matrix} 4 \\ 4/3 \end{matrix}$

$+ \quad + \quad - \quad -$

$f'(-1) \quad 0 \quad f'(1) \quad 4/3 \quad f'(1.5) \quad 2 \quad f'(3) \quad 4 \quad f'(5)$

$\nearrow \quad \nearrow \quad \searrow \quad \searrow \quad \nearrow +$

ene 28-14:14

$\boxed{\begin{array}{l} \text{Crec}(-\infty,0) \cup (0,4/3) \cup (4,\infty) \\ \text{Dec}(4/3,2) \cup (2,4) \\ \text{Max}(4/3,-9/2) \\ \text{Min}(4,-1/2) \end{array}}$

CURV + PI

$y'' = \frac{(6x-16)(x^2-2x)^{\cancel{2}} - (3x^2-16x+1)\,2(\cancel{x^2-2x})(2x-2)}{(x^2-2x)^{\cancel{2}\,4}}$

ene 28-14:18

$\frac{6x^3-12x^2-16x^2+32x-12x^3+76x^2-128x+64}{(x^2-2x)^3}$

$\frac{-6x^3+48x^2-96x+64}{(x^2-2x)^3}$

$D(f \cap f' \cap f'') = \mathbb{R} - \{0,2\}$

$f''(x)=0 \quad -3x^3+24x^2-48x+32=0$

ene 28-14:21

8) d) $y = xe^x$

M. y E

$D(f \cap f') = \mathbb{R}$

$f'(x) = 1 \cdot e^x + xe^x = (1+x)e^x$

$f'(x)=0 \quad x=-1$

$- \quad -1 \quad +$

$f'(-2) \quad -1 \quad f'(0)$

Dec $(-\infty,-1)$

Crec $(-1,\infty)$

$\boxed{\text{Min}(-1, f(-1)) = (-1, \frac{-1}{e})}$

ene 28-14:26

CURVATURA

$D(f \cap f' \cap f'') = \mathbb{R}$

$f''(x) = 1 \cdot e^x + (1+x)e^x = (2+x)e^x$

$f''(x) = 0 \quad x = -2$

$f''(-3)$ (−) -2 $f''(0)$ (+)

Concava $(-\infty, -2)$

Convexa $(-2, \infty)$

PI $(-2, f(-2)) = (-2, \frac{-2}{e^2})$

ene 28-14:27

8) $y = \ln(x+1)$

$D(f): x+1 > 0 \quad x > -1 \quad D = (-1, \infty)$

$f'(x) = \frac{1}{x+1} \quad D(f') = \mathbb{R} - \{-1\}$

$f'(x) = 0$ ∄

-1 (+) $f'(0)$

Creci $(-1, \infty)$

Curv. PI

$y'' = \frac{-1}{(x+1)^2} \neq 0$

-1 (−) $f''(0)$

Concava $(-1, \infty)$

ene 28-14:29

EXT. RELATIVOS

$f'(x_0) > 0 \rightarrow f$ crec en $x = x_0$

$f'(x_0) = 0$

MODO 1 f''

$f''(x_0) > 0$ mínimo

$f''(x_0) = ?$

$f''(x_0) < 0$ Máximo

MODO 2

m M ??

$f'(x_0) < 0 \rightarrow f$ decrecinte en $x = x_0$

ene 28-14:32

PROBLEMAS DE OPTIMIZACIÓN

(Maxim ó minimizar)

$f(x,y) = \ldots$ Optimizar

Relación entre x e y

$f(x)$ Optimizar ⇒

Hallar Max y Min

$f'(x) = 0$

Comp

Mod 1 (f'')

Mod 2 (?)

ene 28-14:34

PAG 289 ① y ②

① $x > 0$

$x + \frac{25}{x}$ sea mínima

$f(x) = x + \frac{25}{x}$ MINIMO

$D = \mathbb{R} - \{0\}$ $D = \mathbb{R} - \{0\}$

$f'(x) = 1 - \frac{25}{x^2}$ $x = \pm 5$

Comprobar

$-$ $+$

0 5 $f'(1)$ $f'(6)$

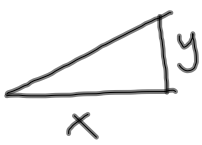
Mino.

SOL: NUMERO ES 5

ene 28-14:37

② $x + y = 10\,cm$

Area maxima.





$A = \frac{1 \cdot 9}{2} = 4{,}5\,cm^2$

$A = \frac{8 \cdot 2}{2} = 8\,cm^2$

$A(x,y) = \frac{xy}{2}$ Max

$x + y = 10$ $y = 10 - x$

$A = \frac{x(10-x)}{2} = \frac{1}{2}(10x - x^2)$

MAX

ene 28-14:39

$D(A) = \mathbb{R}$

$A' = \frac{1}{2}(10 - 2x)$ $D(A') = \mathbb{R}$

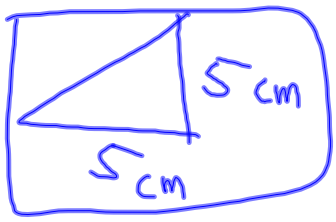
$A' = 0$ $x = 5$

Comp

$+$ $-$

0 5 $A'(1)$ $A'(6)$

Max



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1, 2 Verdes (Resueltos)

3, 4 Abajo

$x^y \cdot y^x = 1$

$\ln(x^y \cdot y^x) = \ln 1$

ene 28-14:45