① $\int \sqrt{(x+3).5}\,dx = \sqrt{5}\int (x+3)^{1/2}\,dx$

$\sqrt{5}\,\frac{(x+3)^{3/2}}{3/2}+C = \frac{2}{3}\sqrt{5(x+3)^3}+C$

② $\int \frac{x+\sqrt{x}}{x^2} = \int \frac{1}{x}+x^{-3/2}\,dx$

$\ln|x| + \frac{x^{-1/2}}{-1/2}+C = \ln|x| - \frac{2}{\sqrt{x}}+C$

Comprob

$\frac{1}{x} - \frac{0.\sqrt{x} - x.\frac{1}{2\sqrt{x}}}{x} = \frac{1}{x} \to \frac{1}{x\sqrt{x}}$

$\frac{\sqrt{x}+1}{x\sqrt{x}}\,\frac{\sqrt{x}}{\sqrt{x}} = \frac{x+\sqrt{x}}{x^2}$

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$\int \frac{x\,dx}{\sqrt{x^2+5}} = \frac{1}{2}\int 2x\,(x^2+5)^{-1/2}\,dx$

$\frac{1}{2}\,\frac{(x^2+5)^{1/2}}{1/2}+C = \sqrt{x^2+5}+C$

$\int \frac{x^2}{\sqrt[3]{1+2x}}$

$1+2x=t^3 \to 2dx = 3t^2\,dt$

$\int \frac{x^2}{\sqrt[3]{t^3}}\,\frac{3t^2\,dt}{2} = \frac{3}{2}\int x^2 t\,dt$

$x = \frac{t^3-1}{2}$ $\quad \frac{3}{2}\int \left(\frac{t^3-1}{2}\right)^2 t\,dt$

$\frac{3}{2}\int \left[\frac{t^6-2t^3+1}{4}\right] t\,dt$

$\frac{3}{8}\int t^7-2t^4+t\,dt$ $\quad t = \sqrt[3]{1+2x}$

$\frac{3}{8}\left[\frac{t^8}{8}-2\frac{t^5}{5}+\frac{t^2}{2}\right]+C$

$\frac{3}{64}\sqrt[3]{(1+2x)^8} - \frac{3}{20}\sqrt[3]{(1+2x)^5} + \frac{3}{16}\sqrt[3]{(1+2x)^2}+C$

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$\int \frac{1}{\sqrt{4-x^2}}\,dx$

$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \operatorname{arcsen} f(x)$

$\frac{1}{\sqrt{4}}\int \frac{1}{\sqrt{\frac{4-x^2}{4}}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dt$

$= \operatorname{arcsen}\frac{x}{2}+C$

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$\int \sqrt{9-4x^2}\,dx$

$\int \sqrt{a^2-b^2x^2}\,dx$

$a=3 \quad b=2$

$\frac{2x}{3} = \operatorname{sen} t \Rightarrow t = \operatorname{arcsen}\frac{2x}{3}$

$bx = a\operatorname{sen} t$

$2x = 3\operatorname{sen} t$

$2\,dx = 3\cos t\,dt$

$\int \sqrt{9-9\operatorname{sen}^2 t}\,\frac{3\cos t\,dt}{2}$

$\int 3\cos t\,\frac{3\cos t}{2}\,dt = \frac{9}{2}\int \cos^2 t\,dt$

$\frac{9}{2}\int \frac{1+\cos 2t}{2}\,dt = \frac{9}{2}\left[\int \frac{1}{2}\,dt + \frac{1}{2}\int \cos 2t\,dt\right]$

$\frac{9}{2}\left[\frac{1}{2}t+\frac{1}{4}\operatorname{sen} 2t\right]+C$

$\frac{9}{4}t+\frac{9}{8}\operatorname{sen} 2t+C$

$\frac{9}{4}\operatorname{arcsen}\frac{2x}{3}+\frac{9}{8}\operatorname{sen}\left(2\operatorname{arcsen}\frac{2x}{3}\right)+C$

mar 19-13:27

$$\int \frac{x+3}{\sqrt{9-x^2}}\,dx = \int \frac{x}{\sqrt{9-x^2}}\,dx + \int \frac{3}{\sqrt{9-x^2}}\,dx$$

Potencias — Arcseno

mar 19-13:31

INTEGRALES RACIONALES $\int \frac{P(x)}{Q(x)}\,dx$

- Logaritmos $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$
- Arcotangente $\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arctg} f(x) + C$
- Potencias $\int f'(x)\,f(x)^n\,dx = \frac{f(x)^{n+1}}{n+1} + C$

mar 19-13:34

Método $\int \frac{P(x)}{Q(x)}\,dx$

[1] GRADO $P(x) \geq$ GRADO $Q(x)$

$P(x)$ | $Q(x)$
$r(x)$ | $c(x)$

$$\int c(x) + \frac{r(x)}{Q(x)}\,dx$$

[2] GRADO $P(x) <$ GRADO $Q(x)$

Factorizar el denominador.

[2.1] Todas las "raíces" reales y simples. (soluciones)

$Q(x) = (x-a)(x-b)(x-c)$

$$\int \frac{P(x)}{Q(x)} = \int \frac{A}{x-a} + \int \frac{B}{x-b} + \int \frac{C}{x-c}$$ LOGARITMOS

[2.2] Todas las raíces reales y alguna no simple

$Q(x) = (x-a)^3(x-b)$

$$\int \frac{P(x)}{Q(x)} = \int \frac{A}{x-b} + \int \frac{B}{x-a} + \int \frac{C}{(x-a)^2} + \int \frac{D}{(x-a)^3}$$

LOGARITMOS — POTENCIAS

[2.3] Alguna raíz no real

$Q(x) = (x-a)(x^2+1)$ (un grado más que en el num)

$$\int \frac{P(x)}{Q(x)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+1}$$

LOGARITMO — LOGARITMO + ARCOTANGENTE

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$$\int \frac{3x^2-7x+4}{2x-3}\,dx$$

$\frac{3x^2}{2x}$ $\frac{-\frac{5}{2}x}{2x}$

$3x^2-7x+4$ | $2x-3$

$-3x^2+\frac{9}{2}x$ | $\frac{3}{2}x-\frac{5}{4}$

$-\frac{5}{2}x+4$

$\frac{5}{2}x-\frac{15}{4}$

$+\frac{1}{4}$

$$\int \frac{3}{2}x - \frac{5}{4} + \frac{\frac{1}{4}}{2x-3}\,dx$$

$$\frac{3}{2}\frac{x^2}{2} - \frac{5}{4}x + \frac{1}{4}\cdot\frac{1}{2}\ln|2x-3| + C$$

$$\boxed{\frac{3}{4}x^2 - \frac{5}{4}x + \frac{1}{8}\ln|2x-3| + C}$$

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$$\int \frac{x^3+4x^2-10x+7}{x^3-7x-6}\,dx$$

$x^3+4x^2-10x+7 \;\big|\; x^3-7x-6$

$-x^3 \quad +7x+6$

$4x^2-3x+13 \qquad 1$

$$\int 1+\frac{4x^2-3x+13}{x^3-7x-6}\,dx$$

$$x+\int \frac{4x^2-3x+13}{x^3-7x-6}\,dx$$

Fact. el den x^3-7x-6

$\begin{array}{c|cccc} -1 & 1 & 0 & -7 & -6 \\ & 1 & -1 & -1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$

$x^2-7x-6=(x+1)(x-3)(x+2)$

$x=\frac{1\pm 5}{2}$ → 3, -2

mar 19-13:45

$$\frac{4x^2-3x+13}{x^3-7x-6}=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{x+2}$$

$4x^2-3x+13=A(x-3)(x+2)+B(x+1)(x+2)+C(x+1)(x-3)$

(x^2-x-6, x^2+3x+2, x^2-2x-3)

MODO 1

$x=-1 \quad 20=-4A \quad A=-5$

$x=3 \quad 40=20B \quad B=2$

$x=-2 \quad 35=5C \quad C=7$

$$x+\int \frac{-5}{x+1}\,dx+\int \frac{2}{x-3}\,dx+\int \frac{7}{x+2}\,dx$$

$$x-5\ln|x+1|+2\ln|x-3|+7\ln|x+2|+C$$

$$x-\ln(x+1)^5+\ln(x-3)^2+\ln(x+2)^7+C$$

$$x+\ln\frac{(x-3)^2(x+2)^7}{(x+1)^5}+C$$

mar 19-13:48

$$\int \frac{x-1}{x^2-2x}\,dx$$

$$\int \frac{3}{4+x^2}\,dx$$

$$\int \frac{(1-x)^2}{1+x}\,dx$$

$$\int \frac{x^4+2x-6}{x^3+x^2-2x}\,dx$$

mar 19-13:55

MODO 2

$4x^2-3x+13=Ax^2-Ax-6A+Bx^2+3Bx+2B+Cx^2-2Cx-3C$

$\left.\begin{array}{l} 4=A+B+C \\ -3=-A+3B-2C \\ 13=-6+2B-3C \end{array}\right\}$ Gauss $\Rightarrow$ $A=-5$, $B=2$, $C=7$

mar 19-13:53