① $\int \cos x + \operatorname{tg} x\,dx = \operatorname{sen} x + \int \frac{-\operatorname{sen} x}{\cos x}\,dx$

$= \operatorname{sen} x - \ln|\cos x| + C$

② $\int \frac{2}{x} + \frac{x}{\sqrt{x}}\,dx = \int \frac{2}{x}\,dx + \int x^{1/2}\,dx$

$2\ln|x| + \frac{x^{3/2}}{3/2} + C = 2\ln|x| + \frac{2}{3}\sqrt{x^3} + C$

③ $\int x\sqrt[3]{2x^2+1}\,dx$

$\frac{1}{4}\int 4x\,(2x^2+1)^{1/3}\,dx$

$\frac{1}{4}\,\frac{(2x^2+1)^{4/3}}{4/3} + C = \frac{\sqrt[3]{(2x^2+1)^4}}{16} + C$

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④ $\int \frac{\operatorname{tg}^2 x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\operatorname{tg}^2 x\,dx$

$\frac{\operatorname{tg}^3 x}{3} + C$

⑤ $\int x \operatorname{arctg} x\,dx$

$u = \operatorname{arctg} x \quad du = \frac{1}{1+x^2}\,dx$

$dv = x\,dx \quad v = \int x\,dx = \frac{x^2}{2}$

$\operatorname{arctg} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\,\frac{1}{1+x^2}\,dx$

$\operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

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$\int \frac{x^2}{1+x^2}\,dx = \int 1 - \frac{1}{x^2+1}\,dx$

$x^2 \;|\; x^2+1 \qquad x - \operatorname{arctg} x + C$

$-x^2-1 \;|\; 1$

$-1$

SOL: $\operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + C$

⑥ $\int \frac{1}{x}\operatorname{sen}(\ln x)\,dx$

$\int f'(x)\operatorname{sen} f(x)\,dx = -\cos f(x) + C$

$-\cos(\ln x) + C$

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⑦ $\int \frac{x}{x^2+4x-21}\,dx$

$x = \frac{-4 \pm 10}{2} \begin{matrix} 3 \\ -7 \end{matrix}$

$\frac{x}{(x-3)(x+7)} = \frac{A}{x-3} + \frac{B}{x+7}$

$x = A(x+7) + B(x-3)$

$x = 3 \quad 3 = 10A \quad A = 3/10$

$x = -7 \quad -7 = -10B \quad B = 7/10$

$\int \frac{3/10}{x-3}\,dx + \int \frac{7/10}{x+7}\,dx$

$\frac{3}{10}\ln|x-3| + \frac{7}{10}\ln|x+7| + C$

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(8) $\frac{1}{4}\int \frac{1}{\frac{3x^2+4}{4}}\,dx$

$\frac{1}{4}\cdot 2\int \frac{1}{\left(\frac{\sqrt{3}x}{2}\right)^2+1}\,dx \quad \frac{\sqrt{3}}{2}$

$\frac{1}{2\sqrt{3}}\arctg\left(\frac{\sqrt{3}x}{2}\right)+C$

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(9) $\int \frac{1+x}{1+\sqrt{x}}\,dx$

$x=t^2 \quad dx=2t\,dt$

$\int \frac{1+t^2}{1+t}\,2t\,dt$

$2\int \frac{t^3+t}{1+t}\,dt$

$t^3+t \;\big|\; t+1$

$-t^3-t^2 \quad t^2-t+2$

$-t^2+t$

$t^2+t$

$2t$

$-2t-2$

$-2$

$2\int t^2-t+2-\frac{2}{1+t}\,dt$

$2\left[\frac{t^3}{3}-\frac{t^2}{2}+2t-2\ln|1+t|\right]+C$

$x=t^2 \Rightarrow t=\sqrt{x}$

$2\left[\frac{\sqrt{x^3}}{3}-\frac{x}{2}+2\sqrt{x}-2\ln|1+\sqrt{x}|\right]+C$

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$\int \cos(\ln x)\,dx$

$u=\cos(\ln x) \to du=-\sin(\ln x)\cdot\frac{1}{x}\,dx$

$dv=dx \Rightarrow v=\int dx=x$

$\cos(\ln x)\cdot x+\int x\frac{1}{x}\cdot\sin(\ln x)\,dx$

$\int \sen(\ln x)\,dx$

$u=\sin(\ln x) \quad du=\cos(\ln x)\cdot\frac{1}{x}\,dx$

$dv=dx \quad v=\int dx=x$

$\sin(\ln x)\cdot x-\int x\cdot\cos(\ln x)\frac{1}{x}\,dx$

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$\int \cos(\ln x)\,dx=\cos(\ln x)\cdot x+\sen(\ln x)\cdot x-\int \cos(\ln x)\,dx$

$2\int \cos(\ln x)\,dx=\cos(\ln x)\cdot x+\sin(\ln x)\cdot x$

$\int \cos(\ln x)\,dx=\frac{\cos(\ln x)\cdot x+\sin(\ln x)\cdot x}{2}+C$

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(11) $f'(x) = \frac{3}{(x+1)^2}$ $f(2)=0$

a) $f(x) = \int f'(x)\,dx$

$\int 3(x+1)^{-2}\,dx = 3\,\frac{(x+1)^{-1}}{-1} + C$

$f(x) = -\frac{3}{x+1} + C$

$f(2)=0$ $-\frac{3}{3} + C = 0$ $C=1$

$f(x) = -\frac{3}{x+1} + 1 = \boxed{\frac{x-2}{x+1}}$

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b) Halle la primitiva de f $(0,1)$

$\int \frac{x-2}{x+1}\,dx = \int 1 - \frac{3}{x+1}\,dx$

$x-2 \;\big|\; x+1$
$-x-1 \quad 1$
$-3$

$x - 3\ln|x+1| + C$

$P(0,1)$ $0 - 3\overset{0}{\ln 1} + C = 1$ $C=1$

$\boxed{x - 3\ln|x+1| + 1}$

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(12) $f'(x) = \begin{cases} 2x-1 & \text{si } x<0 \\ -1 & \text{si } x \geq 0 \end{cases}$

f → sabiendo $f(1)=2$

$\int 2x-1\,dx = x^2 - x + C_1$

$\int -1 = -x + C_2 \Rightarrow$

$f(x) = \begin{cases} x^2 - x + C_1 & \text{si } x<0 \\ -x + C_2 & \text{si } x \geq 0 \end{cases}$

$f(1)=2$ $-1 + C_2 = 2$ $C_2 = 3$

f derivable ⇒ f cont en $x=0$

$\lim_{x\to 0} f(x) \begin{cases} \lim_{x\to 0^-} x^2 - x + C_1 = C_1 \\ \lim_{x\to 0^+} -x + C_2 = C_2 \end{cases}$ $C_1 = C_2$

$f(0) = C_2$

$\boxed{f(x) = \begin{cases} x^2 - x + 3 & \text{si } x<0 \\ -x+3 & \text{si } x \geq 0 \end{cases}}$

$x^2 - x + 3$
$-x$

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(13)

a) Azul ⇒ Recta $\begin{cases} (2,0) \\ (3,3) \end{cases}$

$P(2,0)$
Pte $m = \frac{3-0}{3-2} = 3$

$y - 0 = 3(x-2)$
$\boxed{y = 3x - 6}$
Función

Roja $y = 3$ ⇒ Derivada

b) Recta $P(-1,0)$, $P(0,1)$

$P(-1,0)$
Pte $m = \frac{1-0}{0+1} = 1$

$y - 0 = 1(x+1)$
$y = x+1$ $y' = 1$

Roja $y = -1$ NO

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c). Roja – Parabola

$y = ax^2 + bx + c$

$P(0,-1)$ $-1 = c$

$P(-1,0)$ $0 = a - b + c$ $\quad a - b = 1$

$P(1,0)$ $0 = a + b + c$ $\quad a + b = 1$

$2a = 2 \quad a = 1$

$b = 0$

$y = x^2 - 1$ $\quad y' = 2x$

- Azul recta

$P(0,0)$ $\quad P(0,1)$

$P(1,2)$ $\quad$ Pte $m: \frac{2-0}{1-0}$

$y - 0 = 2(x - 0)$

$y = 2x$

SI

d) NO

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